Modelling the Excess Noise due to Avalanche Multiplication in (Hetero-Junction) Bipolar Transistors

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ABSTRACT. We study the noise behaviour of bipolar transistors for collector voltages close to and beyond the collector-emitter breakdown voltage. We model the excess noise due to amplification of shot noise and due to the impact ionisation itself, both for weak avalanche and for strong avalanche. Our new model accurately predicts the measurements, without the need for parameter fitting to noise data.

I Introduction

Bipolar transistors have been operated for a long time mainly within the limits set by the collector-emitter breakdown voltage BV_{ceo}. The large increase in cut-off frequency has led to a decrease of this breakdown voltage to only a few volts. As a result also the supply voltage has been reduced. Since, however, the turn-on voltage of bipolar transistors has not changed very much the freedom of making circuits within such a low supply voltage has also been reduced. To keep some of the design freedom, more and more designs use a supply voltage larger than BV_{ceo}. This in itself is not a problem, as long as the impedance seen by the base is low enough. The real limiting breakdown is then the junction breakdown voltage BV_{cbo}. Operation above BV_{ceo} *does*, however, lead to large avalanche currents and negative base currents.

Because the avalanche current is much smaller than the collector current—even around and somewhat beyond BV_{ceo}— it has always been assumed that the extra noise as a result of weak avalanche can be neglected. For collector-emitter voltages below BV_{ceo} this is indeed a good assumption. Recently, however, it has been shown experimentally [1] that for voltages close to this breakdown voltage and especially for voltages larger than the breakdown voltage this assumption is no longer true: the minimum noise figure increases notably. It is therefore relevant for circuit designers to have (compact) models that take this excess noise into account. In Ref. [2] a theoretically expression for the minimum noise figure was given based on a simplified equivalent circuit, but no comparison to measurements was done. For a complete model, not only the description of minimum noise figure is important, but also the other three noise parameters, the equivalent noise resistance and the real an imaginary parts of the optimal source impedance, need to be described well. Below we give general model equations for the description of the excess noise due to avalanche that can be implemented into a compact model. Since most compact models take only weak avalanche into account, we will also give expressions in this limit. We will then verify our new model against measurements.

II Theory

II.1 DC model

In Fig. 1 we show a schematic representation of the intrinsic transistor. We show the DC collector current I_{C0} and base current I_{B0} without avalanche and a separate avalanche current I_{avl}. The avalanche current can be expressed in terms of the collector current I_{C0} and multiplication factor M as

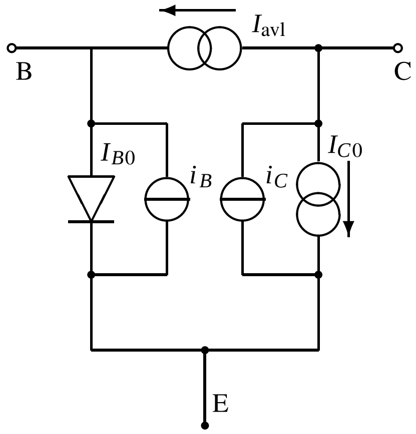

Figure 1: *Equivalent circuit of the intrinsic transistor with the DC collector, base and avalanche currents and the total noise sources* i_C *and* i_B*.*

$$I_{\mathrm{avl}} = (M-1)I_{C0}. \tag{1}$$

For this paper exact expressions for the currents and for the multiplication M are irrelevant and will not be discussed. The total collector and base currents seen at the terminals are now

$$I_C = I_{C0} + I_{\mathrm{avl}}, \tag{2a}$$
$$I_B = I_{B0} - I_{\mathrm{avl}}. \tag{2b}$$

II.2 General noise model

It is important to realise that there are two separate effects that lead to the excess noise. The first effect is solely due to the amplifying nature of impact ionisation. Not only is the DC collector current amplified, also the collector current noise is amplified. This by itself gives a contribution to the total noise. The calculation of this part is rather trivial, since no new noise sources have to be considered. The second effect is the noise of the generated avalanche current itself. In first order this is simply shot noise of the impact-ionisation current. But because the generated noise current is also amplified due to impact ionisation, the noise at the terminals is actually larger than just shot noise.

In the intrinsic transistor we consider three current noise sources: i_{C0} and i_{B0} describe the noise in the collector and base currents before avalanche multiplication, and $i_{\mathrm{I.I.}}$ is the noise due to the randomness of the impact ionisation process. The latter noise source is uncorrelated to the other two, because it is due to a separate physical process. The noise sources i_{C0} and i_{B0} are given by the noise model of the intrinsic transistor without avalanche

[3, 4, 5, 6, 7]. The expression for the noise in $i_{\mathrm{I.I.}}$ has to be determined separately [8, 9].

Our task is to calculate the total noise in collector current and base current, also shown in Fig. 1. The extra noise i_{avl} in the collector current is the sum of the multiplication of the noise in the collector current $(M-1)i_{C0}$ and the noise generated by the impact ionisation $i_{\mathrm{I.I.}}$:

$$i_{\mathrm{avl}} = (M-1)i_{C0} + i_{\mathrm{I.I.}}. \tag{3a}$$

The total collector and base current noise sources are then

$$i_C = i_{C0} + i_{\mathrm{avl}} = M i_{C0} + i_{\mathrm{I.I.}}, \tag{3b}$$

$$i_B = i_{B0} - i_{\mathrm{avl}} = i_{B0} - (M-1)i_{C0} - i_{\mathrm{I.I.}}. \tag{3c}$$

From these it is simple to find the noise current densities:

$$S_{I_C} = M^2 S_{I_{C0}} + S_{I_{\mathrm{I.I.}}}, \tag{4a}$$

$$S_{I_B} = S_{I_{B0}} - 2(M-1)\,\Re\mathrm{e}\, S_{I_{B0}I_{C0}} + (M-1)^2 S_{I_{C0}} + S_{I_{\mathrm{I.I.}}}, \tag{4b}$$

$$S_{I_B I_C} = M S_{I_{B0}I_{C0}} - M(M-1)S_{I_{C0}} - S_{I_{\mathrm{I.I.}}}. \tag{4c}$$

Note that we here explicitely took into account a correlation between intrinsic collector and base current noise sources due to delay effects in the base [4, 5, 6, 7]. In Eq. (4) one can clearly distinguish the two effects mentioned above that lead to excess noise: the effect due to avalanche multiplication of the original noise sources and the extra noise due to the impact-ionisation process. For MOSFETs similar expressions have been given in Ref. [10].

II.3 The noise due to the impact-ionisation process

As mentioned above, the noise due to impact ionisation has a shot-noise-like behaviour in first order. The noise current related to the pair generation, however, also gets amplified due to impact ionisation. The noise at the terminals is therefore larger than just shot noise. It becomes extremely large when the junction is close to junction breakdown, where the multiplication factor diverges. Because of this amplification of the noise, the total noise has to be calculated based on the local value of the ionisation. Fortunately, the resulting noise current can be expressed in terms of the multiplication factor [8, 9]. Note that when non-local avalanche is important the noise is reduced more than what can be calculated from the decrease in the multiplication factor alone [11, 12, 13, 14], and the expressions of Refs. [8, 9] are no longer valid. Since, however, non-local avalanche is as of yet not part of existing compact models, we will disregard the effect on the noise also.

Below, we will use the expressions by McIntyre [8], given for diodes. In our case the initial current I_{C0} contains only electrons (holes do not enter the depletion layer, but they are generated in the depletion layer). The noise in the collector current is then given by

$$S_{I_C} = 2qI_C M^3 \left[1 + (1-k)\left(\frac{M-1}{M}\right)^2\right]. \tag{5}$$

Here k is the ratio between the impact ionisation rate of holes and that of electrons. The expression (5) already contains the multiplication of the noise of the initial current, which here has a value $2qI_{C0}M^2$. The rest is due to the noise from impact ionisation directly:

$$S_{I_{\mathrm{I.I.}}} = 2qI_{C0}M(M-1)[1 + k(M-1)]. \tag{6}$$

For silicon the ionisation factor of holes is much smaller than that of electrons: $k \ll 1$. (Note that this is not the case for all materials.) The expression then reduces to [9]

$$S_{I_{\mathrm{I.I.}}} = 2qI_{C0}M(M-1) = 2qI_{\mathrm{avl}}M. \tag{7}$$

We will use this expression below for simplicity. For other materials the impact ionisation of holes can be close to that of electrons. This leads to larger values of M, but also to larger values of $S_{I_{\mathrm{I.I.}}}$ in terms of M. For the case $k = 1$ this gives

$$S_{I_{\mathrm{I.I.}}} = 2qI_{\mathrm{avl}}M^2. \tag{8}$$

II.4 Explicit formulation

The expressions above are general and hold for any model for the noise of the intrinsic transistor. Next we want to make the results more explicit by expressing the results in terms of transistor currents and charges. For the noise sources without avalanche. we use the results of Ref. [7]

$$S_{I_{C0}} = 2qI_{C0}, \tag{9a}$$

$$S_{I_{B0}} = 2qI_{B0}, \tag{9b}$$

$$S_{I_{B0}I_{C0}} = 2qj\omega Q_{\mathrm{diff,BC}}. \tag{9c}$$

Here $Q_{\mathrm{diff,BC}}$ is the part of the total diffusion charge attributed to the collector side due to charge partitioning. For the noise at the terminals of the intrinsic transistor, we then have, using Eq. (7),

$$S_{I_C} = 2qI_{C0}M(2M-1), \tag{10a}$$

$$S_{I_B} = 2qI_{B0} + (M-1)(2M-1)2qI_{C0}, \tag{10b}$$

$$S_{I_B I_C} = 2qj\omega Q_{\mathrm{diff,BC}}M - 2M(M-1)2qI_{C0}. \tag{10c}$$

II.5 Weak avalanche limit

The avalanche model being used in compact models is limited to weak avalanche, where $M - 1 \ll 1$, or $I_{\mathrm{avl}} \ll I_C$. We find that in practice $M - 1 < 1.1$ is the region where the models have enough accuracy. Because the model for the calculation of the avalanche current is limited to weak avalanche, we only need to take the noise due to avalanche up till the same limit. Taking $M - 1 \ll 1$ in Eq. (10) the expressions simplify to

$$S_{I_C} = 2qI_{C0} + 6qI_{\mathrm{avl}}, \tag{11a}$$

$$S_{I_B} = 2qI_{B0} + 2qI_{\mathrm{avl}}, \tag{11b}$$

$$S_{I_B I_C} = 2qj\omega Q_{\mathrm{diff,BC}} + 2qj\omega Q_{\mathrm{diff,BC}}(M-1) - 4qI_{\mathrm{avl}}. \tag{11c}$$

One can clearly observe the excess noise additions when comparing Eq. (11) to Eq. (9). The expressions given in Eq. (11) are simple to implement in any compact model.

III Experimental verification

For measurements we used a SiGe transistor having a (drawn) emitter size of $0.5 \times 20.3\,\mu\mathrm{m}^2$ and a double-sided base contact from the Philips QUBiC4G process [15]. For this process a fully scaled library is available. RF noise was measured using an Agilent N8975A noise figure meter for a number of pre-characterised source impedances. This allows us to measure a large number of points and determine the accuracy of noise measurements, see *e.g.* Fig. 5. Using open and short de-embedding structures the Y-parameters of the device, measured using an Agilent E8364B vector network analyser, and noise sources are de-embedded to DUT level [16]. Measurement results at a constant collector voltage of $V_{CE} = 2.0\,\mathrm{V}$ have already been presented in Ref. [7].

The simulations are based on the compact model Mextram [17]. The avalanche model of Mextram has been described in more detail in Ref. [18]. We added the excess noise due to impact ionisation using the weak-avalanche result Eq. (11).

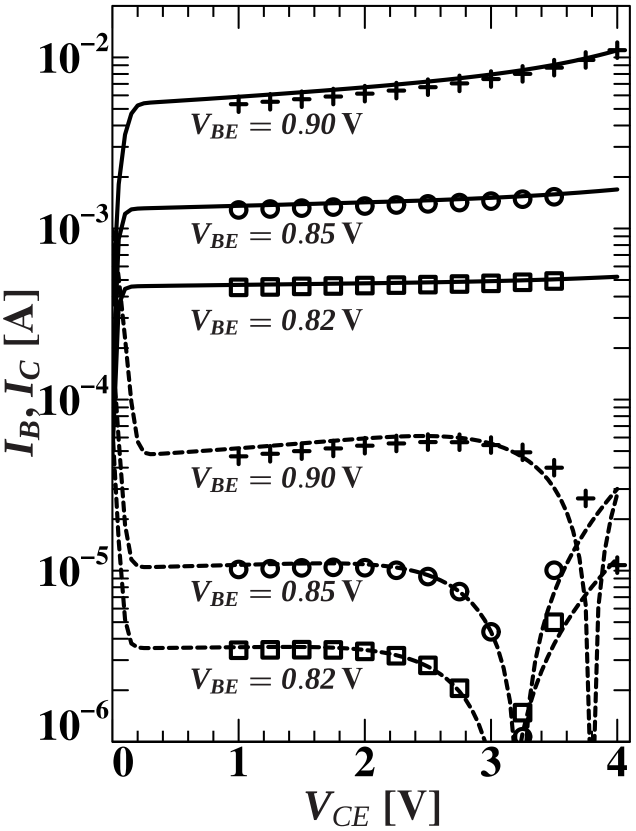

Figure 2: *Collector and base current as function of collector voltage for three values of the base-emitter voltage. Markers are measurements, lines are simulations (solid for I_C, dashed for I_B).*

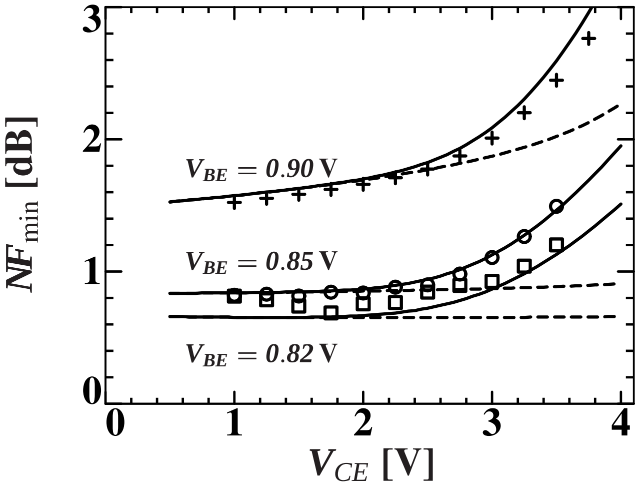

Figure 3: *The minimum noise figure at* $f = 2$ GHz *as function of collector voltage corresponding to the DC measurements shown in Fig. 2. Markers are measurements. Solid (dashed) lines are simulations with (without) taking the excess noise due to avalanche multiplication into account, according to Eq.* (11).

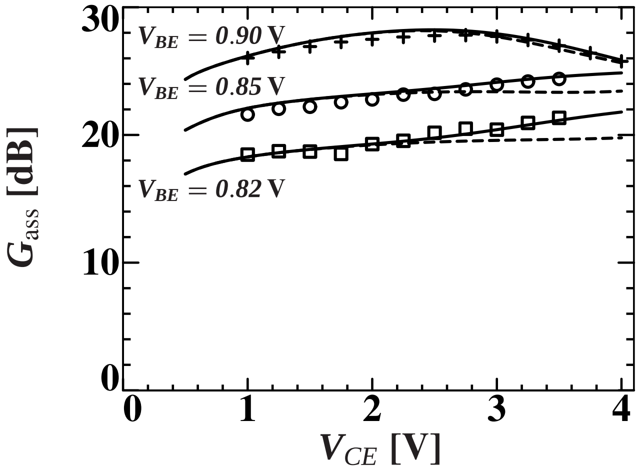

Figure 4: *The associated gain at* $f = 2$ GHz *as function of collector voltage corresponding to the minimum noise figure of Fig. 3.*

In Fig. 2 we show the base and collector currents as function of collector voltage at three values of V_{BE}. Close to $V_{CE} = 4$ V oscillations and snapback effects make it impossible to get reliable measurements for some base-emitter voltages. The breakdown voltage BV_{ceo} is given by the voltage where the base current vanishes. As one can see, the breakdown voltage depends on base-emitter voltage, and is for this transistor around 3 V at low currents. The breakdown voltage increases for higher currents.

For modelling the excess noise due to impact ionisation an obvious prerequisite is that the modelling of the avalanche current is adequate. We therefore optimised the parameters of the avalanche model [18] such that the avalanche current is predicted well both for low and for high current levels. As we show in Fig. 2, we get a good fit. Note that for correct modelling of the currents at higher base-emitter voltage self-heating has to be taken into account.

In Fig. 3 we show the minimum noise figure that corresponds to the DC curves of Fig. 2. We both show measurements and simulations. The minimum noise figure increases with collector voltage. This increase can be observed already for collector voltages below BV_{ceo}. Simulations without the excess noise (dashed lines) show a minimum noise figure that depends almost linearly on collector voltage. When including the excess noise using Eq. (11) the increase in minimum noise figure is modelled accurately. We show the corresponding associated gain in Fig. 4. As one can see, the associated gain is less sensitive to the excess noise, but the effect is not negligible.

We show in Fig. 5 the minimum noise figure as function of frequency for three values of the collector emitter voltage. At low frequency the increase in minimum noise figure due to the excess noise is largest. At higher frequency the minimum noise figure increases due a decrease in current gain. The minimum noise figure is then less sensitive to the collector voltage. Note that without taking the excess noise due to avalanche into account there is almost no V_{CE} dependence of the minimum noise figure (all dashed lines are on top op each other).

To be able to compare our results with that of Greenberg [1] we show in Fig. 6 the minimum noise figure as function of collector current. We also observe the increase in minimum noise figure for increasing collector-emitter voltage. The model predictions are quite accurate, as before. Also in Fig. 6 we show the equivalent noise resistance R_n, whereas in Fig. 7 we show the optimal source impedance Y_{opt}. The results prove that we are not only able to model the minimum noise figure, but also the other three noise quantities. Both the equivalent noise resistance as well as the imaginary part of the optimal source impedance are almost independent of the collector voltage. They are not influenced by the excess noise discussed above either. For the prediction of the real part of the optimal source impedance, on the other hand, we do need to take the excess noise into account (again the dashed lines are all on top of each other). The voltage-dependence of the

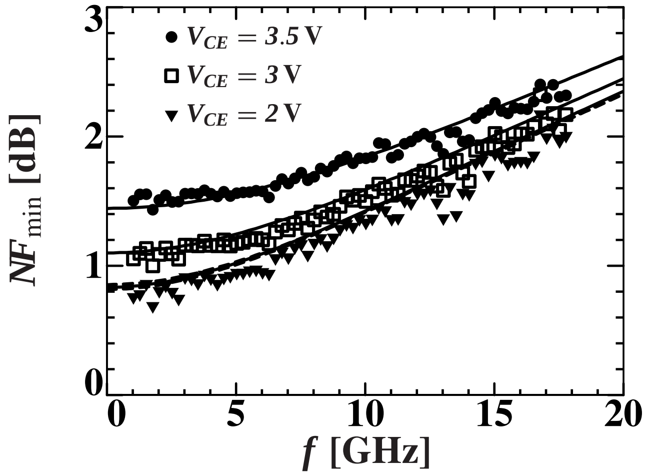

Figure 5: *Minimum noise figure as function of frequency at $V_{BE} = 0.85$ V. Dashed and solid lines are as in Fig. 6 (dashed lines are barely visible).*

real part of $Y_{\rm opt}$ is responsible for the voltage dependence of the associated gain, see Fig. 4.

IV Conclusions

We have shown that for accurate modelling of the noise for voltages close to the breakdown voltages and beyond one needs to take into account the excess noise due to impact ionisation. This excess noise has two contributions: the noise of the intrinsic transistor is multiplied due to impact ionisation and the impact ionisation itself also generates noise. We have shown how to take this effect into account within the compact model Mextram, but it can also be added to other models. The results clearly show an improved modelling of the noise of bipolar transistors at higher collector voltages, without the need for parameter fitting to noise data.

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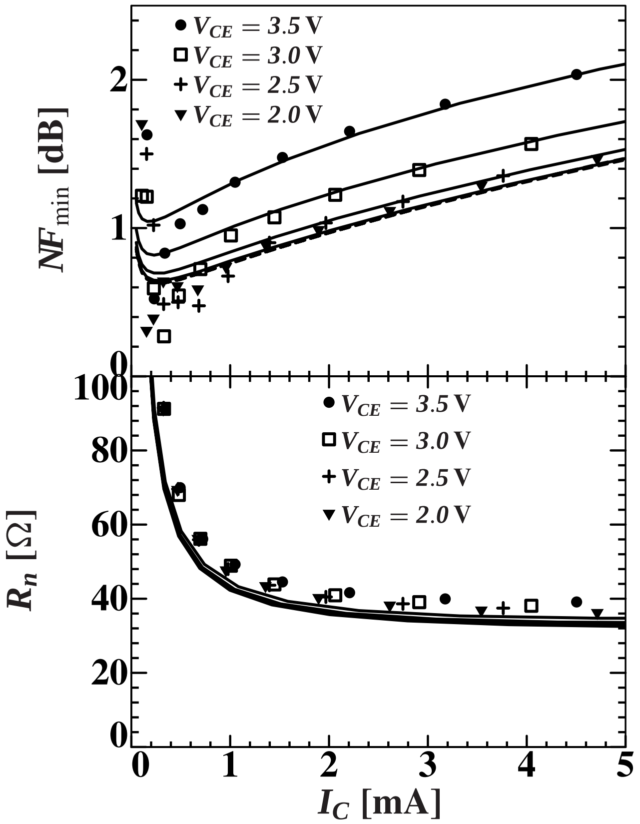

Figure 6: *Minimum noise figure and equivalent noise resistance as function of collector current at $f = 2$ GHz. Markers are measurements. Dashed and solid lines are as in Fig. 3. The dashed lines (barely visible) are all close to the lowest solid line.*

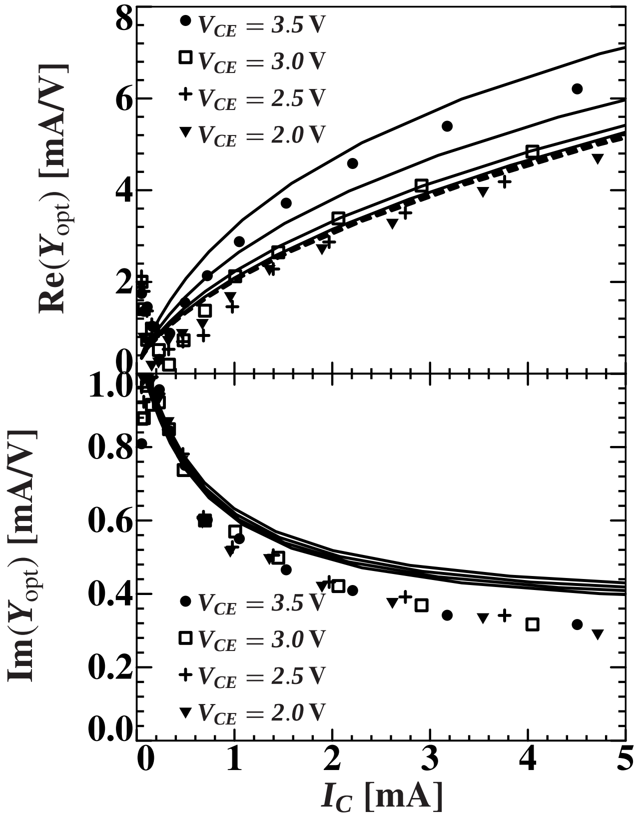

Figure 7: *Real and imaginary part of the optimal source impedance $Y_{\rm opt}$ as function of collector current. Markers are measurements, solid (dashed) lines are simulations with (without) the excess noise, as in Fig. 6.*